BEAM LOADING

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Abstract
Beam loading on RF cavities may seriously limit the performance of high-intensity circular accelerators or storage rings. The RF power requirements to correct for beam loading will be first examined in several typical cases (lepton and hadron machines). Then, the methods to control the RF system (feedback and feedforward) and to achieve stability under heavy beam loading conditions will be reviewed.

1. INTRODUCTION

In accelerator language, beam loading usually refers to the effects induced by the passage of the beam in the radio frequency cavities. As such, it could be considered to be one particular example of the more general problem of the beam interaction with its surroundings, in this case the cavity impedance.

However the beam loading problem deserves a special treatment, for several reasons. Firstly, the RF cavities are very often the largest contributor to the total ring impedance (in the following we shall concentrate on circular machines) and, consequently, power considerations play a very important role in beam-loading problems. Secondly, contrary to many other machine elements, the RF cavities are well known items being carefully designed and measured, from the RF point of view, and are easily accessible from the outside world via the RF power amplifier. Dedicated correction techniques can therefore be used where not only the cavity but also its associated RF amplifier are included.

In the following we shall first consider the stationary situation established in the beam-cavity system, the two extreme cases being when the bunches are wide apart and when every bucket is filled. The case of a long train of bunches followed by a gap is of particular importance in high intensity machines (ion clearing gaps in e^+e^- machines, beam dump gap for hadron colliders, for instance) and will be examined separately. Travelling-wave cavities with their inherent advantages as far as beam loading is concerned will be examined in this context.

Before settling to the stationary situation, the beam-cavity system undergoes a transient phase which may be very harmful to the beam, especially for hadron machines without natural damping. To circumvent this problem, it will be shown that RF power must be available. Finally the various methods used to control the RF amplifier-cavity combination in order to suppress beam-loading effects will be reviewed.

2. SINGLE-BUNCH PASSAGE IN A CAVITY

When the distance between bunches is very large compared to the filling time of the cavity, the fields induced by the previous bunches, or the previous bunch passages of the same bunch, have decayed sufficiently and can be neglected. Consequently, before the bunch passage the RF waveform is a pure sinewave produced by the RF generator (Fig. 1a).

The effect of the bunch passage is to excite an additional field in the cavity (Fig. 1b). For a short bunch (short compared to the RF period) and considering only the fundamental resonance of the cavity, the excited waveform is an exponentially decaying sinewave oscillating at the resonant frequency of the cavity ω_c.

Combining the generator driven and beam driven waveforms, one obtains the total voltage $V(t)$ at the cavity gap (Fig. 1c).

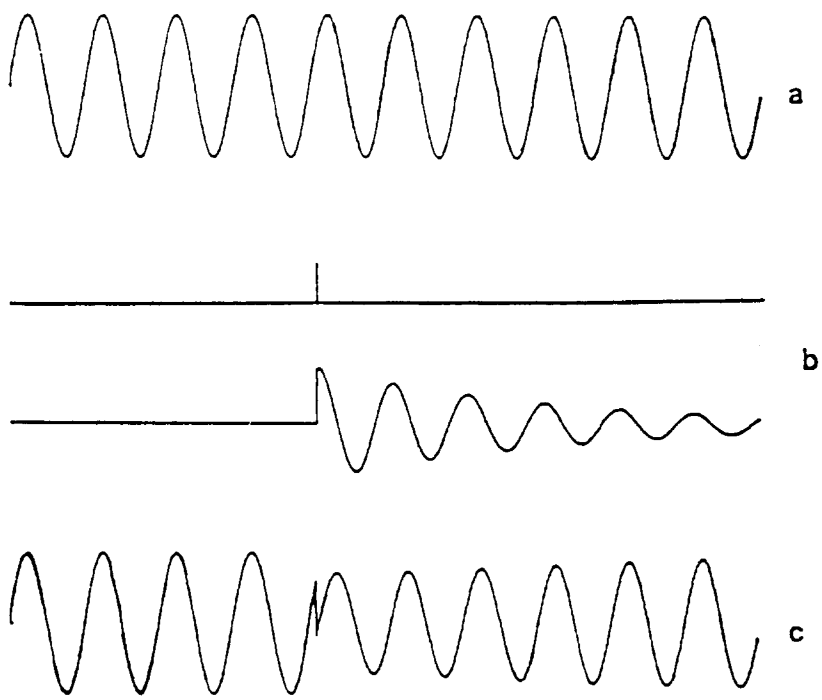


Fig. 1 Single-bunch passage in a cavity

Assuming a negligible beam current (unperturbed voltage), the power delivered to the beam by the RF generator is simply:

$$P = -\frac{\vec{V}_g \cdot \vec{i}_b}{2} \tag{1}$$

where $\vec{V}_g$ is the generator driven voltage and $\vec{i}_b$ is the fundamental component of the beam current.

When crossing the gap the charge q induces the voltage V_{b0}, and loses a fraction of its energy which is finally transformed into heat in the cavity walls before the next bunch passage. In this situation the effective gap voltage for the beam is modified by the effect of the beam. We can define an effective gap voltage V evaluated in the following way.

In the transient phase (assuming a short bunch length compared to the RF period), the cavity gap impedance can be represented by a single capacitor C related to the cavity parameters by:

$$\frac{1}{C} = \frac{R}{Q_0}\omega_c \tag{2}$$

where ω_c is the resonant frequency of cavity, Q_0 the unloaded cavity quality factor and R the shunt impedance of the cavity (circuit convention). Obviously $V_{b0} = q / C$, and the energy lost by the bunch and stored in the cavity just after the bunch passage amounts to:

$$W = \frac{1}{2} C \; V_{b0}^2 = \frac{1}{2} q \; V_{b0} \; . \tag{3}$$

This corresponds to an average power loss of:

$$\frac{W}{T_b} = \frac{1}{2} i_{bDC} V_{b0} = \frac{1}{4} i_b V_{b0}$$

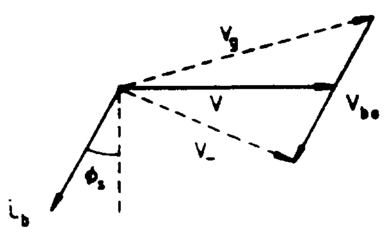
where T_b is the bunch distance and $i_{bDC} = i_b/2$ (short bunch approximation). The net power received by the beam P' is simply, remembering that $\vec{i}_b$ and $\vec{V}_{b0}$ are in phase:

$$P' = -\frac{\vec{V}_g \cdot \vec{i}_b}{2} - \frac{1}{4} \vec{i}_b \cdot \vec{V}_{b0} \tag{4}$$

$$P' = -\frac{1}{2}\left(\vec{V}_g + \frac{1}{2}\vec{V}_{b0}\right) \cdot \vec{i}_b = -\frac{1}{2} \vec{V} \cdot \vec{i}_b \tag{5}$$

Here $\vec{V}$ is the effective RF voltage, delivering the net power P' to the beam.

Equation (5) leads to the vector diagram of Fig. 2, which shows the voltages before ($\vec{V}_g$) and after $(\vec{V}_-)$ the bunch passage and their relations with the bunch current. Obviously the voltage V_g to be delivered by the generator is higher for the same effective voltage V, than in the case of no beam loading. The excess power can be easily computed from the cavity shunt resistance and beam current.



V_g: generator driven voltage

V_-: voltage after bunch passage

V: net voltage experienced by the beam

Fig. 2 Vector diagram – single-bunch passage in a cavity

One can also remark that the beam "sees" only one-half of its own induced voltage:

$$\vec{V}_b = \frac{1}{2} \vec{V}_{b0}$$

where

$$\vec{V} = \vec{V}_g + \vec{V}_b \; .$$

This result is sometimes quoted as "the fundamental theorem of beam loading", and can be demonstrated more generally (P. Wilson [1]) using linearity and superposition. Similarly, it is easy to show that, in fact, V_{b0} represents the sum of all beam induced voltages for all cavity modes.

3. MULTIPLE-BUNCH PASSAGES [1,2]

We look for a stationary solution, when an infinite train of bunches, spaced by h_b RF periods, crosses the cavity gap. Following P. Wilson's analysis [1] we should replace V_g in Eq. (5), which represents the voltage just before the bunch passage, by the combination of the generator-driven voltage and the voltage resulting from all previous bunch passages. The decay of the voltage between two successive bunch passages is simply $\delta = T_b / T_f$ where T_f is the cavity time constant ($T_f = 2Q_L / \omega_c$, Q_L: loaded cavity quality factor), and the phase shift with respect to the RF generator amounts to $\psi = \omega_c T_b - 2\pi h_b$. The relation $\vec{V} = \vec{V}_g + 1/2\vec{V}_{b0}$ will therefore transform into:

$$\vec{V} = \vec{V}_g + \vec{V}_{b0}\left(e^{-\delta}e^{j\psi} + e^{-2\delta}e^{2j\psi} + \ldots\right) + \frac{1}{2}\vec{V}_{b0} = \vec{V}_g + \vec{V}_b \tag{6}$$

in which the term in brackets represents the contributions of all previous bunch passages, whereas the last one reflects the effect of the bunch on itself (Fig. 3).

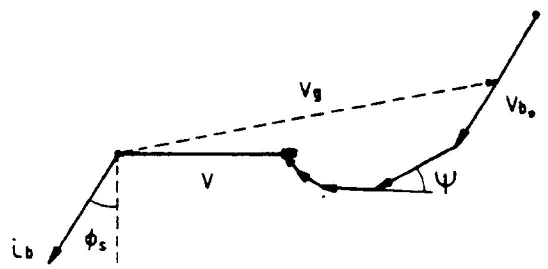


Fig. 3 Vector diagram – multiple-bunch passages

Using the sum of the geometric series:

$$V_{b0}\left(1 + e^{-\delta}e^{j\psi} + e^{-2\delta}e^{2j\psi} + \ldots\right) = \frac{V_{b0}}{1 - e^{-\delta}e^{j\psi}} \tag{7}$$

one obtains:

$$\vec{V}_b = \vec{V}_{b0}\left(\frac{1}{1 - e^{-\delta}e^{j\psi}} - \frac{1}{2}\right) \tag{8}$$

which, when separating real and imaginary parts leads to:

$$V_b = V_{b0}\left[F_1(\delta,\psi) + jF_2(\delta,\psi)\right] \tag{9}$$

with:

$$F_1(\delta,\psi) = \frac{1 - e^{-2\delta}}{2\left(1 - 2e^{-\delta}\cos\psi + e^{-2\delta}\right)}$$
$$F_2(\delta,\psi) = \frac{e^{-\delta}\sin\psi}{1 - 2e^{-\delta}\cos\psi + e^{-2\delta}} \tag{10}$$

If we introduce now the more usual cavity parameters:

$$\tan\phi_c \text{ (detuning angle)} = 2Q_L \frac{\omega_c - \omega}{\omega_c} \tag{11}$$

$$\beta \text{ (coupling coefficient);} \quad Q_L = Q_0 \frac{1}{1+\beta} \tag{12}$$

and $\delta_0 = T_b / T_{f0}$ (T_{f0} being the filling time of the unloaded cavity), Eq. (9) becomes:

$$V_b = 2i_0 R\delta_0 \left[F_1(\beta,\phi_c) + jF_2(\beta,\phi_c)\right] \tag{13}$$

where i_0 is the DC beam current.

The functions F_1 and F_2 are given by:

$$F_1(\beta,\phi_c) = \frac{1 - e^{-\delta_0(1+\beta)}}{2D} \tag{14}$$

$$F_2(\beta,\phi_c) = \frac{e^{-\delta_0(1+\beta)} \sin \delta_0(1+\beta) \tan \phi_c}{D} \tag{15}$$

$$D = 1 - 2e^{-\delta_0(1+\beta)} \cos\left[\delta_0(1+\beta)\tan\phi_c\right] + e^{-2\delta_0(1+\beta)} \; . \tag{16}$$

From these expressions, it is possible to calculate the generator power needed to produce a given accelerating voltage V. For a generator which is assumed to be matched, by using, for instance, a circulator between generator and cavity, one obtains [1]:

$$P_g = \frac{(1+\beta)^2}{4\beta} \frac{V_g^2}{2R} \frac{1}{\cos^2\phi_c} \{A^2 + B^2\} \tag{17}$$

where A and B are complicated functions of cavity and beam parameters [1]. Numerical computations are required to optimize the various parameters in order to minimize P_g.

4. LIMITING CASE $\delta_0 \cong 0$

When the bunch distance T_b is short compared to the unloaded cavity filling time (Fig. 4), Eqs. (10) simplify to:

$$F_1(\delta_0,\beta) = \frac{-1}{\delta_0(1+\beta)(1+\tan^2\phi_c)} \tag{18}$$

$$F_2(\delta_0,\beta) = \frac{-\tan\psi}{\delta_0(1+\beta)(1+\tan^2\phi_c)} \; . \tag{19}$$

Combined with (13), one obtains:

$$V_b = \frac{i_b R}{1+\beta} \, \frac{1}{1 - j\tan\phi_c} \; . \tag{20}$$

In this case the cavity gap waveform is approximately sinusoidal (Fig. 4), and the equivalent circuit of Fig. 5, where the beam current is represented by its component at the RF frequency i_b can be used. There the coupling coefficient β is simply related to the cavity and generator shunt resistances by: $\beta = R / R_g$. Obviously, V_b given by Eq. (20) is the cavity voltage (sinusoidal in the approximation $\delta_0 \cong 0$) developed when i_g (generator current) = 0.

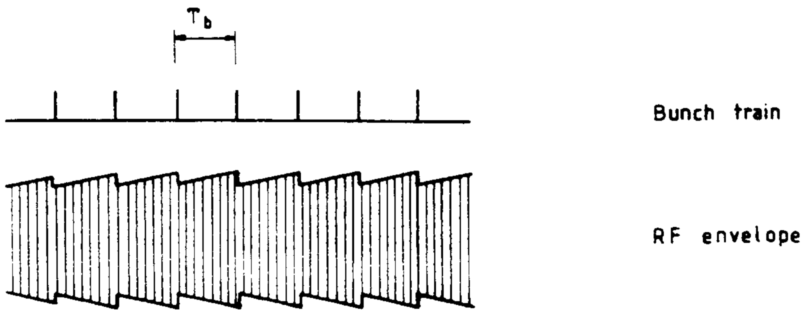


Fig. 4 Case $\delta_0 \cong 0$. The RF waveform is a quasi sinusoid

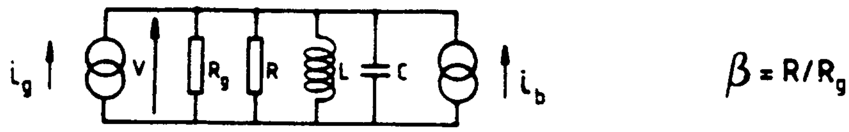


Fig. 5 Equivalent circuit for the case $\delta_0 \cong 0$

In the vector diagram of Fig. 6a, the total current $\vec{i}_t = \vec{i}_g + \vec{i}_b$ drives the RLC circuit and produces the gap voltage V. For a given V, the vector $\vec{i}_t$ follows the dotted line in Fig. 6a, when the detuning angle ϕ_c is varied. This is because the admittance of the equivalent RLC circuit has a constant real part.

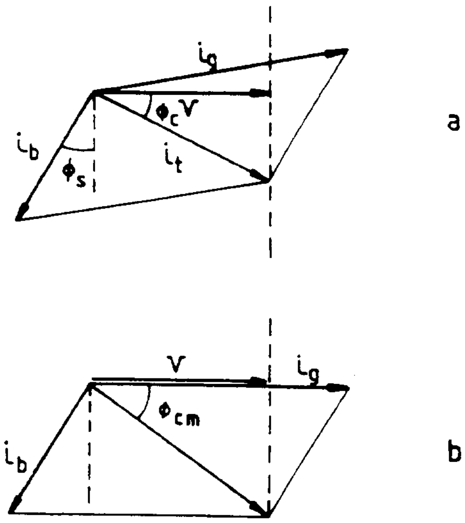


Fig. 6 Vector diagrams for the case $\delta_0 \cong 0$. Optimum tuning in (b)

The required RF power:

$$P_g = \frac{1}{2}\vec{V} \cdot \vec{i}_g \tag{21}$$

is a minimum for given V, i_b and ϕ_s, if the two conditions:

$$\tan\phi_{c0} = -i_b \frac{R\cos\phi_s}{(1+\beta)V} \tag{22}$$

$$\beta_0 = 1 + i_b \frac{R\sin\phi_s}{V} \tag{23}$$

are fulfilled. The minimum RF power for $\phi_c = \phi_{c0}$ and $\beta = \beta_0$ is given by:

$$P_{gm} = \frac{V^2}{2R} + Vi_b \sin\phi_s \ , \tag{24}$$

the first term corresponding to the cavity losses and the second to the power delivered to the beam. The optimum phase condition ($\phi_c = \phi_{c0}$) corresponds to i_g and V being in phase (Fig. 6b). Usually there is a servo-tuner which measures the phase difference between RF drive and gap voltage, and controls the cavity tune via a mechanical tuner or ferrite bias, for instance. At equilibrium of the servo-tuner, Eq. (22) is automatically satisfied. It corresponds to a cavity detuning:

$$\Delta\omega_0 = \omega_c - \omega = \frac{1}{2}\frac{R'}{Q_L}\omega_c i_b \frac{\cos\phi_s}{V} = \frac{1}{2}\frac{R}{Q}\omega_c i_b \frac{\cos\phi_s}{V} \tag{25}$$

where R' is the equivalent resistance of R and R_g in parallel.

On the contrary, the cavity coupling is usually fixed by construction, and can only be optimized for a given value of i_b and ϕ_s. However for a hadron storage ring, where $\phi_s = 0$, the critical coupling ($\beta = 1$) corresponds to the optimum situation.

For very large machines and high beam currents, RF cavity detuning may become equal to or larger than the revolution frequency f_{rev}. When cavity resonance coincides with a revolution harmonic one can expect large modulations of the RF voltage if all RF buckets are not equally populated (see § 5), but also a strong coupled bunch instability driven by the fundamental RF cavity impedance.

The critical cavity parameter here is $\frac{R}{Q} \cdot \frac{\omega_c}{V}$ (equal to the charge stored on the equivalent C of the cavity) which should be minimized if such effects are to be avoided. For this purpose wide aperture cavities are interesting (small R/Q), especially superconducting (large V, without sacrificing Q). Another solution is to increase the stored energy (and consequently the equivalent stored charge) with additional resonators coupled to the accelerating cavity [3].

5. THE CASE OF MISSING BUNCHES

We assume again $\delta_0 \cong 0$, but consider a gap in the train of bunches, corresponding to a number of consecutive buckets being empty. Beam loading is, in this case, periodic at the revolution frequency and will result in a periodically modulated RF waveform.

We shall try to find directly the periodic equilibrium situation in the very simple case $\phi_s = 0$ (no acceleration) and a lossless cavity ($R' = \infty$). In the rotating frame at ω, the RF voltage vector $\vec{V}$ evolves under the effects of: a) cavity detuning $\Delta\omega$, b) beam current (we assume here that the generator current is unmodulated at f_{rev} harmonics).

Cavity detuning produces a phase variation $d\varphi_1$ of $\vec{V}$:

$$d\varphi_1 = \Delta\omega \;\; dt \tag{26}$$

Beam current $i_b(t)$ produces an additional quadrature $(\phi_s = 0)$ cavity voltage

$$dv = \frac{1}{2}\frac{R}{Q}\omega \;\; i_b(t)dt \tag{27}$$

The total phase variation:

$$d\varphi = \left(\Delta\omega - \frac{1}{2}\frac{R}{Q}\omega \frac{i_b(t)}{V} \right) dt \tag{28}$$

integrated over one turn must be zero for a stationary situation. As the amplitude of V remains constant (only quadrature contributions) it follows:

$$\Delta\omega_0 = \frac{1}{2}\frac{R}{Q}\frac{\omega}{V}\frac{1}{T}\int_0^T i_b(t)dt = \frac{1}{2}\frac{R}{Q}\frac{\omega}{V}\bar{i}_b \tag{29}$$

where T is the revolution period and $\bar{i}_b$ the average RF component of the beam current.

One obtains again Eq. (25) as expected, from which the phase variation of the RF voltage can be expressed as:

$$\varphi(t) = \int_0^t d\varphi = \frac{1}{2}\frac{R}{Q}\frac{\omega}{V}\int_0^t \left(\bar{i}_b - i_b(t)\right)dt \tag{30}$$

In the simple, but very useful case of a continuous beam of constant intensity, with a gap of length $(T - t_0)$, one finds a linear phase variation (Fig. 7) with a maximum excursion:

$$\Delta\varphi_{\max} = \frac{1}{2}\frac{R}{Q}\frac{\omega}{V}\bar{i}_b(T - t_0) = \Delta\omega_0(T - t_0) \tag{31}$$

Interestingly enough Eq. (31) shows no resonance even if $\Delta\omega = 2\pi / T$ as one would expect. This is because the AM modulation of the beam current is exactly compensated, for the side band falling on the cavity resonance, by the phase modulation given by Eq. (31). This result can also be obtained in a more complete way by frequency analysis [4, 5]. For optimum detuning $\Delta\omega_0$ one obtains:

$$\frac{p_v}{a_b} = -\frac{\Delta\omega_0\left(s + \sigma - \Delta\omega_0 \tan^3\phi_s\right)}{(s + \sigma)^2 - \Delta\omega_0^2 \tan^2\phi_s} \tag{32}$$

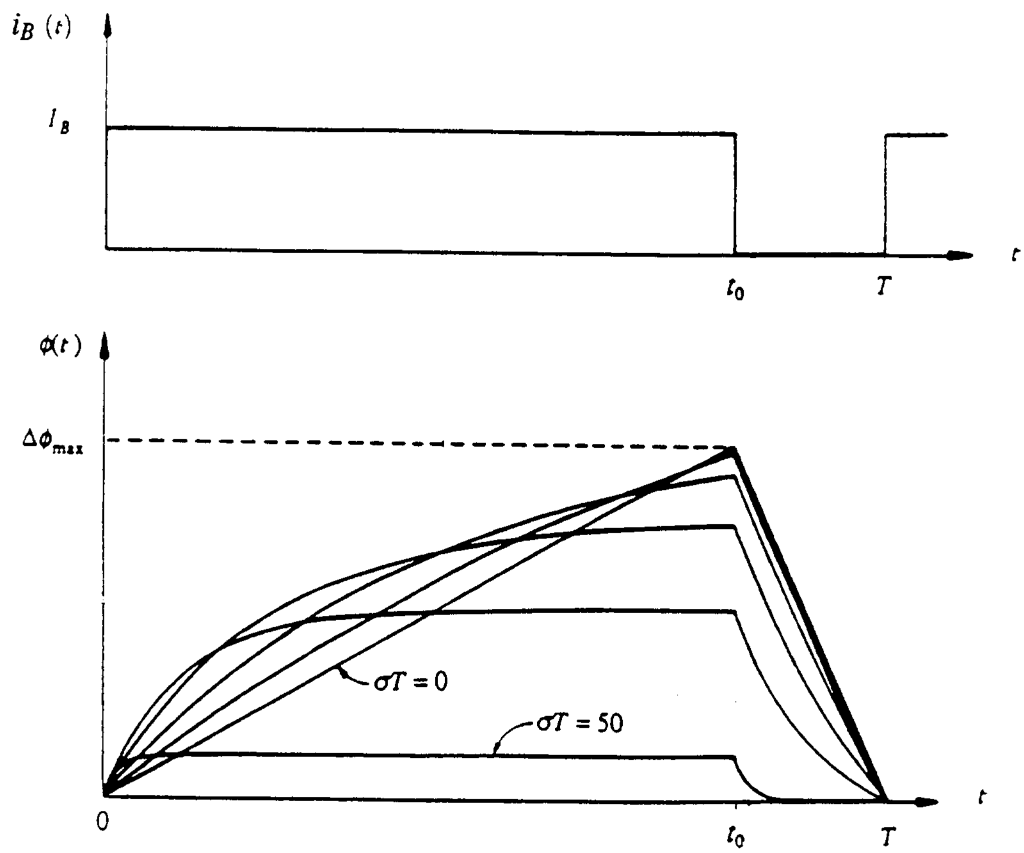


Fig. 7 Phase modulation of V_{RF} due to a gap in the bunch train for various values of the cavity bandwidth

$$\frac{a_v}{a_b} = -\frac{\Delta\omega_0 (s + \sigma + \Delta\omega_0 \tan\phi_s) \tan\phi_s}{(s+\sigma)^2 - \Delta\omega_0^2 \tan^2\phi_s} \tag{33}$$

where p_v, a_v and a_b are the phase and amplitude modulation indices of V and a_b the beam modulation amplitude due to the gap. s is the Laplace variable and $\sigma = \omega_c / 2Q_L$ the half cavity bandwidth. In the case $\phi_s = 0$, the amplitude modulation of V disappears; only remains the phase modulation given by:

$$\frac{p_v}{a_v} = -\frac{\Delta\omega_0}{s+\sigma} \tag{34}$$

and illustrated in Fig. 7 (low pass filter characteristics).

Phase modulation along the batch may have important consequences, in particular in the case of colliding beams: the azimuthal position of the beam crossing point depends on the bunch positions in the trains and may become excessive [4, 5]. Here again a cavity with a large stored energy (small $\Delta\omega_0$) is interesting. Alternatively one can also increase the cavity bandwidth (in particular with RF feedback, section 8.3), but at the expense of more RF power.

6. THE CASE OF A TRAVELLING-WAVE STRUCTURE

It is known that in a long chain of coupled resonators travelling waves can propagate within some frequency limits, i.e. passbands of the structure. In the travelling mode of operation, the structure is terminated by its characteristic impedance and behaves like a transmission line (Fig. 8). At synchronism, the phase velocity v_φ of the wave equals the particle velocity v_p, giving maximum voltage seen by the beam, like an RLC circuit at resonance.

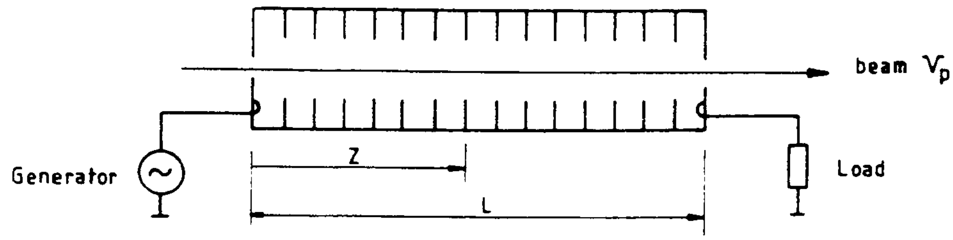


Fig. 8 Schematic of a travelling-wave structure

For a single-bunch passage, it is usually possible to neglect the cavity coupling as the energy transfer from cell to cell is much slower than the bunch velocity $(v_g << v_p;\ v_g\text{: group velocity})$. The previous analysis can therefore be applied to the quasi uncoupled resonators. It is generally applied also for the standing-wave mode of operation of multicell cavities, which are non-terminated structures [6]. However, for a repetitive train of many bunches, the RLC equivalent circuit model would fail in the travelling-wave mode because the waves excited by previous bunch passages also propagate along the structure.

For instance, at exact synchronism $(v_\varphi = v_p)$, the waves excited in each cell by the beam passage add linearly in the forward direction, and, on average, cancel in the reverse direction, in a forward travelling-wave structure. In other words, the decelerating electric field E_z is simply proportional to the distance along the structure counted from the feed point.

If the synchronism is not perfect, we must introduce a phase factor exp $j(\omega t - \beta_\varphi z)$ for each individual wave, where $z = v_p t$ and β_φ is the wave propagation constant, with the result that the induced field $E_z(z)$ is proportional to the integral:

$$E_z(z) \sim \int_0^z \exp\ j(\omega t - \beta_\varphi z)dz = \int_0^z \exp\ j\theta dz\ . \tag{35}$$

Expanding θ around the synchronous point (ω_0, β_0) one obtains:

$$\theta = \omega t - \beta_\varphi z = \left(\frac{\omega_0 + \delta\omega}{v_p} - \beta_0 - \Delta\beta\right)z = \left(\frac{\delta\omega}{v_p} - \Delta\beta\right)z\ . \tag{36}$$

Introducing $v_g = \delta\omega / \Delta\beta_\varphi$ and the phase slip angle τ defined by:

$$\tau = L\frac{\delta\omega}{v_g}\left(1 - \frac{v_g}{v_p}\right) \tag{37}$$

L being the structure length, one obtains:

$$\theta = -\tau \frac{z}{L} \tag{38}$$

and:

$$\int_0^z \exp\, j\theta \; dz = \frac{1-\exp\left(-j\frac{\tau}{L}z\right)}{j\frac{\tau}{L}} \, . \tag{39}$$

In particular, for $z = 0$, the integral vanishes: the beam induced field is zero on the upstream end of the structure (generator side). This is a very important result as it shows that, for a travelling-wave structure, there is no beam loading effect seen by the RF generator, which always remains matched without the need for a circulator. In the case of a backward-wave structure, where the generator is connected to the downstream end of the structure, this result is still valid. Beam loading only changes the field on the load side: not all the generator power goes into the load, some fraction is transferred to the beam.

The total voltage V_b seen by the beam is obtained by integrating the electric field, given by (39) along the structure:

$$V_b \sim \int_0^L \frac{1-\exp\left(-j\frac{\tau}{L}z\right)dz}{j\frac{\tau}{L}} \, . \tag{40}$$

It gives finally:

$$V_p = i_b R_2 \frac{L^2}{8}\left[\left(\frac{\sin \tau/2}{\tau/2}\right)^2 - 2j\frac{\tau - \sin\tau}{\tau^2}\right] \tag{41}$$

where the proportionality factor R_2, called the series impedance of the structure, is characteristic of its geometry [7]. Figure 9 shows a plot of Eq. (41) in the complex impedance plane.

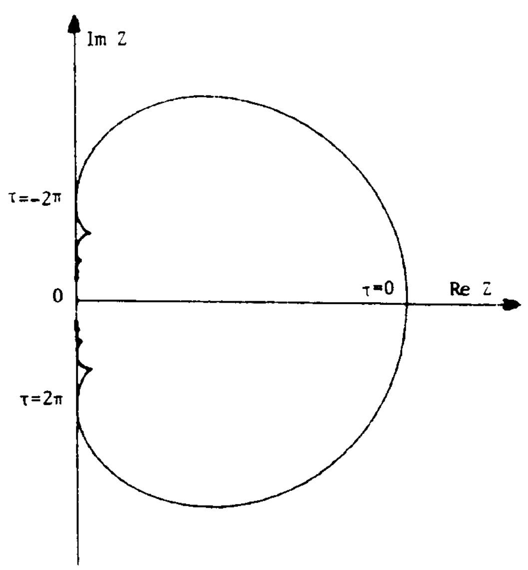


Fig. 9 Impedance seen by the beam of a travelling-wave structure

7. TRANSIENT CORRECTION

Consider again the case of a cavity represented by its RLC equivalent circuit. Even in the case $\delta_0 \cong 0$ (quasi sinusoids) the stationary solution of Section 4, where only the RF frequency component is considered, cannot describe transient situations, when V or i_b change rapidly.

The worst case situation corresponds to a sudden change of $\vec{V}$ (e.g. transition) or $\vec{i}_b$ (injection of a prebunched beam, fast ejection of part of the beam). The resulting unwanted transient must of course be damped for the stationary solution described above to settle down properly, but it must also be short compared with the synchrotron period T_S. This condition will ensure that the effects on the beam such as mismatch and subsequent blow-up, or even loss of particles, will be minimum, or in other words that beam loading will be properly corrected.

We shall now consider the example of a prebunched beam i_b injected into an empty machine. Before injection the servo-tuning keeps $i_t = i_g$ and V in phase. Immediately after injection the new vector i_b destroys the equilibrium, and V changes by a large amount until the tuning loop retunes the cavity to a different value. Unless one uses very fast tuners, which may lead to multiloop stability problems [8], it will take more than a small fraction of a synchrotron period for the tuning loop to settle at its new value, the result being a strong distortion of the longitudinal phase plane.

The only way to maintain V constant during the transient phase of the tuner is to act via the RF power generator which provides a fast control of V. The obvious solution (Fig. 10) is to change i_g into i_g' when the beam is injected. If we make:

$$i_g' = i_g - i_b \tag{42}$$

the total current in the cavity does not change and, at constant tuning, V stays constant.

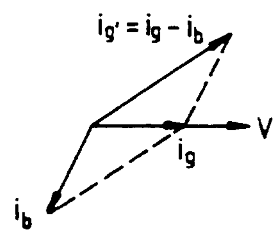


Fig. 10 Correction of beam-loading transient with the power generator

In the simple case of no acceleration, the amplitude of the peak current i_g' which must be delivered by the RF power tube during the transient phase of the tuner, is given by:

$$|i_g'|^2 = |i_g|^2 + |i_b|^2 \tag{43}$$

or, for a high beam loading

$$i_g' \cong i_b \; .$$

The RF generator must be able to deliver the current i_b and sustain the voltage V. This determines the required *installed* power $\left(\frac{1}{2} V i_b\right)$ for transient beam loading correction, and more precisely the RF tube characteristics (max V, max i_g') and the transformation ratio between tube and cavity.

In the case of a matched generator (Fig. 11) with a circulator inserted between RF power and cavity, the additional current means also additional *real* power. In the $\phi_s = 0$ case, the peak power $\hat{P}$ required during the transient is given by [4]:

$$\hat{P} = \frac{Z_0}{8}\left(V^2\left(\frac{1}{Z_0}+\frac{1}{R}\right)^2 + \left(\frac{V}{x} - i_b(t)\right)^2\right) \tag{44}$$

where Z_0 is the line impedance transformed to the gap and x is the reactive part of the cavity impedance

$$x = \frac{1}{2}\frac{R}{Q}\frac{\omega_0}{\Delta\omega} \quad .$$

Assuming $\Delta\omega = 0$ before injection, one can optimise $\hat{P}$ by adjusting the transformation ratio according to:

$$\frac{1}{Z_0^2} = \left(\frac{i_b}{V}\right)^2 + \frac{1}{R^2} \tag{45}$$

One obtains in this case, and for heavy beam loading ($R\ i_b >> V$):

$$\hat{P} \cong \frac{V i_b}{4} \tag{46}$$

Remark that the installed power (which is also real power here) is only half of what would be required without circulator.

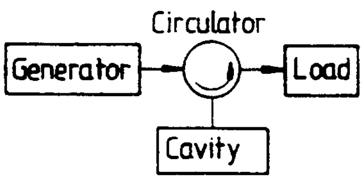


Fig. 11 A circulator to match the RF power generator

However, the correction described above corresponds to the worst case situation. In certain cases it is possible to minimize the required peak power or peak current. In particular, by pretuning the cavity before injection, one can make the two powers, before and after injection, equal and obtain in this case $\hat{P} = |V||i_b|/8$ (for $\Phi_s = 0$) again with a circulator between power source and cavity. One can also reduce the transient on i_b with multiple injections of smaller currents, or by adjusting the bunching factor of the injected beam.

In the above analysis, we assumed that all bunches are submitted to the same RF voltage. This may not be true in the case of unequal filling of the ring which will give a modulation of V at f_{rev} and its multiples. The same analysis applies here: at each "batch" passage transient beam loading must be corrected to make all bunches see the same RF voltage. This effect is particularly important in large machines not only at injection but also at transition. As before, condition (44) is valid in the worst case situation, i_b being now the batch current [4, 9].

8. RF DRIVE GENERATION

During the transient phase of the tuner, we must synthesize i_g' to meet condition (42) and correct for the effect of beam loading at injection or at transition. The same is true in the case of uneven filling of the ring. It obviously implies that i_g' (or the corresponding power $\hat{P}$) is available from the RF generator, otherwise transient beam loading cannot be corrected completely. Various techniques used to generate the proper i_g' will now be examined. As will become apparent in the following, they are closely related to the stability of the entire RF system.

8.1 Amplitude and phase servo loops

The synthesis of i_g in order to keep V constant irrespective of the beam loading can be done with two servo loops (Fig. 12): the first acting on the amplitude of i_g (amplitude loop) controls $|V|$, and the second maintains the relative phase of V and i_b constant through the control of the phase of i_g (phase loop). The cut-off frequency f_c of the loops must be much larger than the synchrotron frequency f_s, which means very strong damping of beam oscillations. This justifies the simplified stability analysis [8] in which the beam transfer function is neglected. The cut-off frequency f_c is obviously limited by the delays in the system, including the cavity bandwidth, but more fundamentally by the revolution frequency f_{rev}. The simple configuration of Fig. 12 with high loop gains cannot correct transient beam loading at f_{rev} and its multiples.

Steady beam loading with its associated cavity detuning could excite mode $n = 0$ (Robinson instability [10]) if it were not heavily damped by the phase loop. However, mode $n = 1$ (one wavelength per turn) which is not damped may show up also due to cavity detuning and must be suppressed by dedicated feedback circuitry acting through the RF cavity itself.

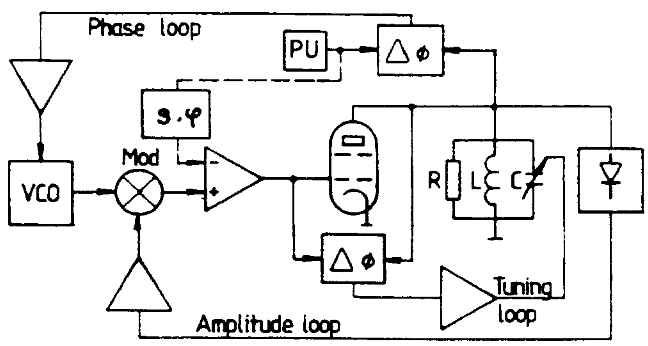


Fig. 12 Tuning, amplitude and phase loops. Feedforward correction (dotted line).

Independent amplitude and phase control of V is a well known technique for proton machines. It works satisfactorily for relatively small beam currents, i.e. when the gap voltage is predominantly determined by the generator current (typically $|i_b| < |i_g|$). For higher beam currents, a variation of the amplitude of i_g, for instance, not only results in a variation of the amplitude of V but also of its phase. In other words, the two loops, which were independent at low beam currents, become coupled together and an unstable behaviour of the system results above a certain beam current threshold. Pedersen's detailed analysis [8], confirmed by experiments on the CERN PS booster, leads to the generalized Robinson stability criterion, valid for $\Phi_s = 0$, optimum detuning, σ large, and assuming a negligible beam response in the frequency range of interest:

$$\frac{|i_b|}{|i_g|} < \sqrt{2 + \frac{f_a}{f_T} + \frac{f_T}{f_a} + \frac{f_p}{f_T} + \frac{f_T}{f_p} + \frac{f_a}{f_p} + \frac{f_p}{f_a}} \tag{47}$$

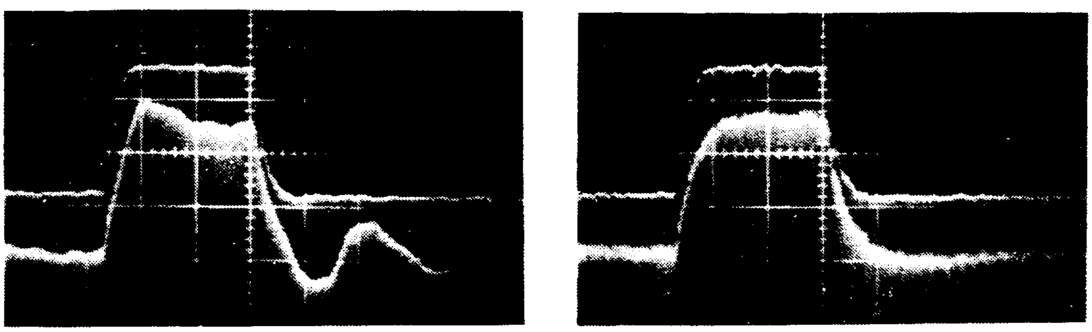
where f_a, f_p and f_T are the unity gain frequencies of the loops (amplitude, phase and tuning respectively). Although the threshold is weakly dependent on the loop cut-off frequencies, it might be dangerous in this configuration to increase the servo-tuner bandwidth.

In the more general case, no analytical simple stability criterion could be found. Simulations [15] have shown that in almost all cases if the quantity $|i_b|/|i_g|$ is kept below 2 stability of the RF system is ensured with some safety margin. A safe design criterion for a new RF system would be $|i_b|/|i_g| < 1$ which can be expressed by the simple rule of thumb: beam induced voltage not larger than RF voltage.

Although it is in principle possible to compensate loop coupling by an additional decoupling circuitry so increasing the instability threshold, much simpler solutions are offered by feedforward correction and RF feedback.

8.2 Feedforward correction

With a pick-up electrode followed by a filter centred at f_{RF}, one can obtain a signal proportional to $-i_b$ independently from the RF system, and generate i_g' (RF drive with beam) according to (32) with a simple adder. Applied to the amplitude and phase servo loops described in section 7.1, the method consists of injecting into the input of the RF amplifier the pick-up signal, with proper amplitude and phase (g, φ), to generate the $-i_b$ current at the gap (Fig. 12). The amplitude and phase loops now act on the quantity i_g, corresponding to no beam loading, instead of i_g', and the cross couplings between loops are removed, as can be shown analytically and experimentally [11] (Fig. 13). As a result, the instability threshold can be considerably increased and, for instance, stable operating conditions have been observed in the CERN PS for $|i_b|/|i_g| \cong 8$ to 10.



(a) (b)

Fig. 13 Transient response of amplitude loop with (b) and without (a) feedforward correction (CERN PS machine). The loop response becomes oscillatory at high intensity (bottom trace) without feedforward correction

The signal corresponding to $-i_b$ does not need to be synthesized with the ultimate precision as it only removes the loop couplings and restores stability. For a varying RF frequency, the pick-up to cavity delay must be continuously adjusted, and the variations in gain and phase of the RF power amplifier (assumed linear) corrected.

Feedforward correction can also be considered as a means to reduce the effective impedance of the cavity seen by the beam. At the RF frequency, the beam induced voltage on the cavity amplifier combination is zero for a perfect correction. From this point of view, high amplitude and phase loop gains at f_s are no longer required to correct beam loading as V is automatically kept constant by the feedforward compensation. Application of this technique (low loop gains) was, for instance, used on the Brookhaven AGS during adiabatic capture.

It is interesting to mention a variant of the feedforward technique derived from the Alvarez linear accelerator technology. If the generator is a gridded tube (tetrode or triode), its output impedance is high if maximum RF power is to be extracted from the tube. When connected to the cavity by a long line, it fully reflects the beam loading wave travelling from the cavity to the generator. One can choose the length of the line to make the reflected wave cancel the beam induced voltage at the gap, the high impedance of the generator is then transformed into a quasi-short circuit at the cavity. Note that, even with no voltage induced on the gap, the generator sees a mismatched load with beam and must be able to deliver the current under this condition. This technique is in use on the CERN PS 200 MHz RF system, with trombones inserted on the feeder lines of the fixed-tune cavities.

If the pick-up to cavity delay is adjusted to be exactly one turn (T), beam loading cancellation can be achieved, not only at f_{RF}, but also at frequencies $f_{RF} \pm nf_{rev}$. This is relatively easy at fixed RF frequency, for example in the CERN ISR [12], but with modern sampled or digital filters and variable delays it is also possible to follow a varying RF frequency. The overall result is a rapidly changing impedance, ideally zero at frequencies nf_{rev}, but twice as large at intermediate frequencies, $(n+1/2)f_{rev}$, where there are no beam current components (Fig. 14). With a one turn delay and perfect cancellation, the voltage perturbation only lasts T which is small compared with T_S since $Q_s = (f_{rev}T_s)^{-1}$ is usually << 1. In other words the reduction of the magnitude of the cavity impedance at the synchrotron satellites $nf_{rev} \pm mf_S$ is also large (factor $(2\sin m\pi Q_s)^{-1}$) for a small Q_S.

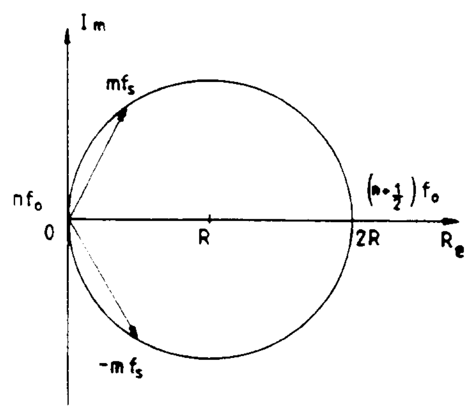


Fig. 14 Residual impedance at synchrotron sidebands for a one turn delay feedforward correction

8.3 RF feedback

We can consider the cavity itself as a beam pick-up tuned at f_{RF} and obtain the $-i_b$ signal from the gap itself. This leads to the configuration of Fig. 15 in which one obviously recognizes a feedback loop built around the RF power amplifier. From the loop equations one obtains:

$$i_g' = i_g - \frac{GZ\ i_b}{1+GZ} \tag{48}$$

which, for $GZ >> 1$ (GZ: loop gain, Z cavity impedance) reduces to Eq. (32):

$$i_g' \cong i_g - i_b$$

i_g being here the generator current with no beam.

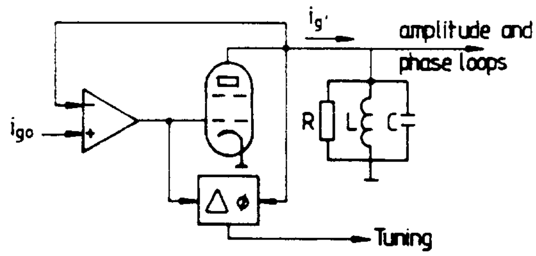


Fig. 15 RF feedback around the power amplifier

The feedback loop automatically generates the correct compensating signal, which is another way of saying that it keeps the controlled parameter V constant. One can consider RF feedback as a means to reduce the output impedance of the RF amplifier, a well known design being the cathode follower with its low output impedance which shunts the cavity.

Even simpler, but of limited efficiency, is the use of a triode instead of a tetrode as the RF power tube, the internal plate to grid feedback reducing the output impedance. In the same way pulsing the DC current of the RF tube or powering a second tube, in parallel [13], has been used to reduce the output impedance of the RF amplifier for short periods.

In the case of Fig. 15, the cavity parameters (pole at $f_{RF}/2Q_L$) and the total delay of the feedback path determine the loop stability. The preamplifiers which are selected for the shortest propagation delay must be located very close to the power amplifier-cavity combination. As an example Table 1 gives the parameters for the CERN PS booster second harmonic system, operating between 6 and 16 MHz [14].

Table 1
Feedback parameters of the CERN PSB second-harmonic system

Preamplifier gain	25 x
Bandwidth	150 MHz
Power	130 W (1 dB compression)
Propagation delay	5 ns
Impedance reduction factor	21 dB at 6 MHz
	14.5 dB at 16 MHz

For a varying RF frequency one could adjust the delay of the return path to keep the 180° phase condition at f_{RF}. However, in many designs, for instance the second harmonic PS booster and the future PS RF system, a wide bandwidth preamplifier is used to keep the total delay short enough to ensure stability over the entire RF frequency range, without programming the phase. In this case it is extremely important to damp the higher resonances of the cavity or to reject the corresponding signals in order to avoid parasitic oscillations of the feedback system at high frequencies.

The RF feedback technique is very attractive since it reduces the effective impedance of the cavity not only at the RF frequency but also over a large bandwidth. This feature is particularly helpful to avoid self-bunching instabilities in storage rings for debunched beams and was used at the CERN ISR and AA for instance.

In such a feedback system the total phase slip should be less than about $\pm\pi/4$ over the unity gain bandwidth $2\,\Delta\omega$ of the open loop system, giving the condition [15]:

$$\Delta\omega = \pi / 4\tau \tag{49}$$

where τ is the overall delay in the feedback path. For a fixed tuned cavity and a small detuning angle, the cavity impedance (RLC approximation) far from the ω_c resonance is given by $Z = R / 2jQ_L(\Delta\omega / \omega_c)$. The overall loop gain, GZ, at the $\pm\Delta\omega$ points is of the order of unity: this gives an upper limit for GZ and a minimum value of the impedance seen by the beam, $R_{\min}$, given by:

$$R_{\min} \cong \frac{2}{\pi}\tau\frac{R\omega_c}{Q} = 4\frac{R}{Q}f_{\mathrm{RF}}\tau \tag{50}$$

Equation (39) shows that the ultimate performance of wideband RF feedback only depends on τ and the cavity geometry (R/Q parameter).

When a gap is present in the circulating beam, RF feedback will automatically correct for it and, in the limit of very high gain suppress phase and amplitude modulations given by Eqs. (32) and (33). This may be undesirable if the corresponding RF power is not available. In this case, one may program in phase the RF reference (i_g in Fig. 14) as the expected phase production and greatly reduce the power requirements [4].

If a servo tuner is used in conjunction with RF feedback, it may be necessary to control it by the normalized reactive power of the amplifier [16] instead of the classical phase detection between RF drive and cavity voltage.

RF feedback is now widely used in high intensity RF systems [5, 17-20]. The dynamic properties and stability of the RF system are critically dependent on the delay of the power amplifier. This requirement has led, in particular to the development of short delay high power klystrons for such applications. RF feedback opens the way to using superconducting cavities also in high intensity applications. As a practical advantage, RF feedback can easily be set up without the presence of beam, contrary to the feedforward technique; it finally makes the behaviour of the RF system far less sensitive to the beam current.

8.4 The RF feedback with long delay [5, 21]

In large RF systems, the CERN SPS for instance, long delays may be unavoidable and the conventional RF feedback would have a too restricted bandwidth, much smaller than the cavity bandwidth itself in the SPS case. Transient beam loading at multiples of f_{rev} would not be corrected, leading to phase oscillations of fractions of the beam and possible coupled bunch instabilities.

In order to solve the problem, we observe that a large gain G is only needed in the vicinity of the revolution frequency harmonics where beam current components exist. Outside these bands, the phase rotation due to the excessive delay will be unimportant if G can be made small enough. With a return path transfer function having a comb-filter shape with maxima at every f_{rev} harmonic, this condition can be satisfied. In addition, the overall delay of the system must be extended to exactly one machine turn (T) to ensure a zero phase at the $f_{\mathrm{RF}} + nf_{\mathrm{rev}}$ frequencies.

The comb filter transfer function (Fig. 16) is of the form:

$$H(j\omega) = \frac{G_0}{1 - K\exp(-j\delta\omega T)} \tag{51}$$

where G_0 and K are constants ($0 < K < 1$).

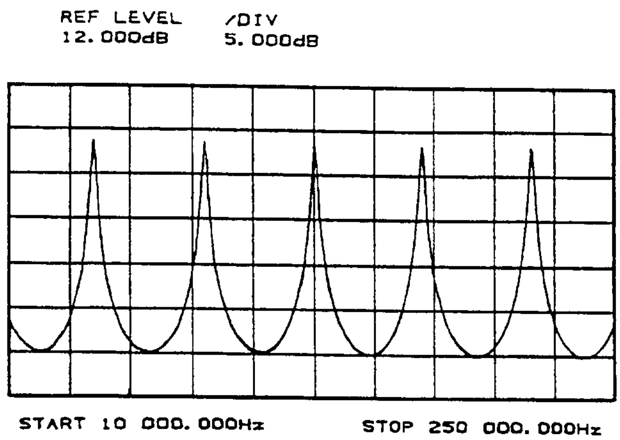


Fig. 16 Comb-filter transfer function $K = 7/8$, $N = 462$

Combined with the one turn delay (transfer function: $\exp(-j\delta\omega T)$), the overall open loop transfer function becomes:

$$G(j\omega)Z(j\omega) = \frac{G_0 Z(j\omega)}{\exp(j\delta\omega T) - K} \tag{52}$$

represented in the complex plane by a circle for a slowly varying $Z(j\omega)$ as shown in Fig. 17. The complex plane origin is encircled and therefore the gain of the system is limited by the stability condition. In the vicinity of the cavity resonance, where Z is maximum and real (note that for a travelling wave structure Z is always real [4], the circle crosses the negative real axis at a distance $-G_0 Z / (1 + K)$ from the origin.

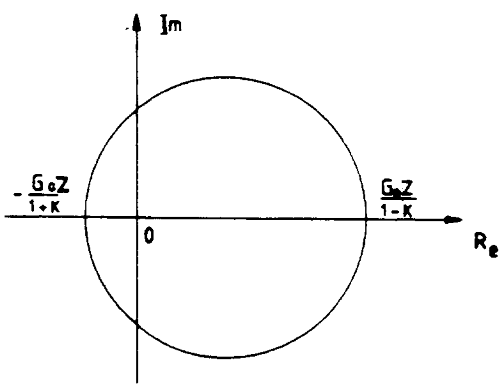


Fig. 17 Open-loop transfer function for RF feedback with long delay

Stability obviously requires that $|G_0 Z| < 1 + K$, and it can be shown that this condition is also sufficient even outside resonance for an RF cavity approximated by a single RLC equivalent circuit.

For Z real, the apparent impedance of the cavity Z':

$$Z' = Z \frac{\exp(j\delta\omega T) - K}{\exp(j\delta\omega T) - K - G_0 Z} \tag{53}$$

is real for frequencies:

$$f_{\mathrm{RF}} + nf_{\mathrm{rev}}; \quad Z' = Z\frac{1-K}{1-K+G_0Z} << Z \tag{54}$$

and:

$$f_{\mathrm{RF}} + \left(n+\frac{1}{2}\right)f_{\mathrm{rev}}; \quad Z' = Z\frac{1+K}{1+K-G_0Z} \tag{55}$$

To stay at a reasonable distance from the stability limit, take for instance $G_0Z = (1+K)/2$. This gives, at frequencies $f_{\mathrm{RF}} + \left(n+\frac{1}{2}\right)f_{\mathrm{rev}}$, $Z' = 2Z$ as in the case of feedforward correction, whereas for the revolution frequency harmonics one obtains:

$$Z' \cong Z(1-K) \tag{56}$$

for $(1 - K) << 1$.

By making K close to unity, RF feedback approaches the theoretical performance of the feedforward correction but with all the inherent advantages of closed loop systems; in particular no critical adjustments are needed. Similarly, the time response of the RF feedback is entirely determined by the one turn delay as in the feedforward case. Note that the unity gain frequency of the servo in this case is of the order of $f_{\mathrm{rev}}/2$.

The residual impedance at the synchrotron sidebands is approximately the same as for a one turn delay feedforward correction (for $K \cong 1$ and $G_0Z \cong 1$); its phase changes sign at each nf_{rev} harmonic resulting in a rotation of the complex synchrotron frequency shift curve. The coupled-bunch, cavity-driven, instability thresholds must be obtained numerically [18].

Except for relatively small machines with fixed RF frequency, long delay feedforward or feedback techniques could only be envisaged with the help of modern signal processing technology, i.e. sampled or digital filters. The digital comb filter is derived from the well known first-order low-pass recursive filter shown in Fig. 18. With a sampling frequency Nf_{rev} locked to a subharmonic of the RF frequency, the theoretical bandwidth of the filter is $Nf_{\mathrm{rev}}/2$, corresponding to $N/2$ maxima in the comb filter response (N = 462 in the SPS design). Implementation of the one-turn delay is straightforward in digital technology with a memory (R.A.M. or first-in first-out type), or with more modern arithmetic processors.

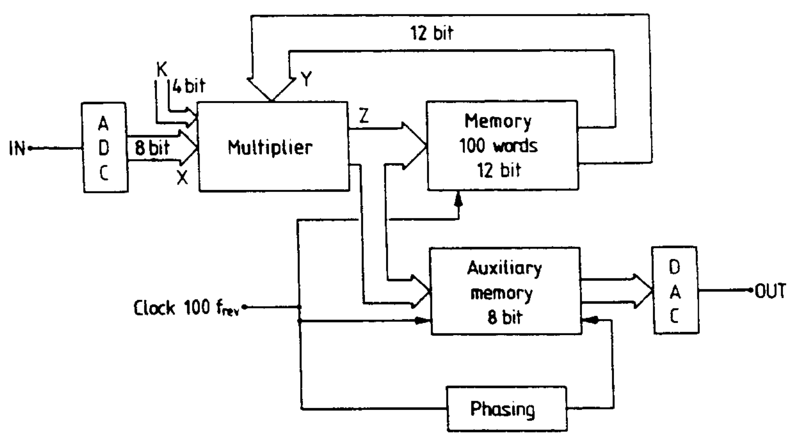


Fig. 18 The digital filter and delay

It is also possible to combine short and long delay RF feedbacks on the same cavity. The additional impedance reduction at the f_{rev} harmonics is however limited to factors 3 to 5, because of the non-linear phase response of the cavity with short delay feedback. The SPS superconducting cavities [19] are now equipped with combined RF feedback circuits to cope with the highest beam intensities.

In e^+e^- machines where Q_S is not so small, one can also use more complex periodic filters, the objective being to reduce the equivalent cavity impedance preferentially at the synchrotron sidebands $f_{rev} \pm f_s$ to avoid coupled bunch instabilities [5].

The speed of the various elements, limited by the cycle time (*T/N*), may become very critical requiring the fastest A-D converters (flash converters), memories and multipliers (parallel multipliers).

The RF signals may have to be translated in frequency to be conveniently processed. Coherent mixing with separate channels for in-phase and in-quadrature components is necessary to reject the unwanted image frequencies (measured rejection > 35 dB), and to make the overall electronic chain look a linear system. For a varying RF frequency the correct phase can even be maintained with an artificial delay inserted between the output and input local oscillators as in Fig. 19.

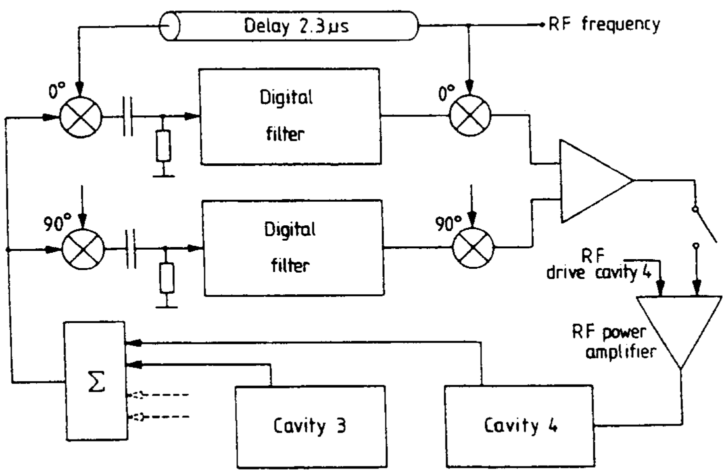


Fig. 19 Layout of the SPS RF feedback system

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